8/29/25 Lecture 3

Def - An $m \times n$ matrix is an array of #'s $\begin{cases} 1 \le i \le m \\ 1 \le j \le n \end{cases}$

Def - Matrices A + B are <u>row equivalent</u> if A can be transformed by elementary row operations into B

MW we learned that

- <u>Thm</u> - Every matrix is row equivalent to a matrix in Reduced Row Escelon Form (RREF)
  - Proof: We learned an algorithm for accomplishing Thm.

    "Gauss - Jordan elimination" or "row reduction"

If A is a matrix, then Gauss - Jordan elimination is an algorithm that uses row operations to put A in RREF. It has two main parts:

I. Working from top to bottom, do the following with each row of A.

a) Swap with a row below if necessary so that the pivot is as far left as possible.

b) Add multiples of the row to each row underneath so that all entries below the pivot become $\emptyset$. (referring to the row we moved down in part a)

II. Working from bottom to top, do the following with each row of A:

a) divide the pivot entry to make it equal 1

b) add multiples to rows above to make entries above the pivot equal to $\emptyset$.

After I. get a matrix in eschelon form

II. get a matrix in RREF.

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix} + \left(-\tfrac{1}{2}\right) \cdot R_1 \rightarrow \begin{bmatrix} 2 & -1 & 1 \\ 0 & \tfrac{1}{2} & \tfrac{5}{2} \end{bmatrix} \cdot 2 \rightarrow \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 5 \end{bmatrix} + R_2$$

eschelon form

$$\rightarrow \begin{bmatrix} 2 & 0 & 6 \\ 0 & 1 & 5 \end{bmatrix} \cdot \tfrac{1}{2} \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \end{bmatrix}$$

RREF

This matrix is Rank 2

Def - The rank of matrix A is the # of pivots in a RREF matrix $\tilde{A}$ row equivalent to A.

- The rank can at most be the minimum number of rows or columns.
  - Ex: 2×3 matrix can have Rank 2.

- A (column) n-vector $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ is an $n \times 1$ matrix

- A row n-vector is a $1 \times n$ matrix.

- You can add matrices of the same size and multiply by constants

Ex Adding

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ -1 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Ex multiply

$$5\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & -5 & 5 \\ 5 & 0 & 15 \end{bmatrix}$$

Many of usual rules for arithmetic apply.

$$A+B = B+A$$

$$(b+c)A = bA + cA$$

$$\boxed{c(A+B) = cA + cB}$$

Proof: Write $A = (a_{ij})$, $B = (b_{ij})$

Then $c(A+B) = c\big((a_{ij}) + (b_{ij})\big) = c(a_{ij} + b_{ij}) = \boxed{(ca_{ij} + cb_{ij})}$

AND

$$cA + cB = c(a_{ij}) + c(b_{ij})$$

$$= (ca_{ij}) + (cb_{ij})$$

$$\boxed{= ca_{ij} + cb_{ij}}$$

Match!

<u>Def</u> → If $\vec{v}$ and $\vec{w}$ are in $\mathbb{R}^2$ are vecs then the <u>dot product</u> of $\vec{v} \cdot \vec{w}$ is:

$$\vec{v} \cdot \vec{w} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} = v_1w_1 + \ldots + v_nw_n$$